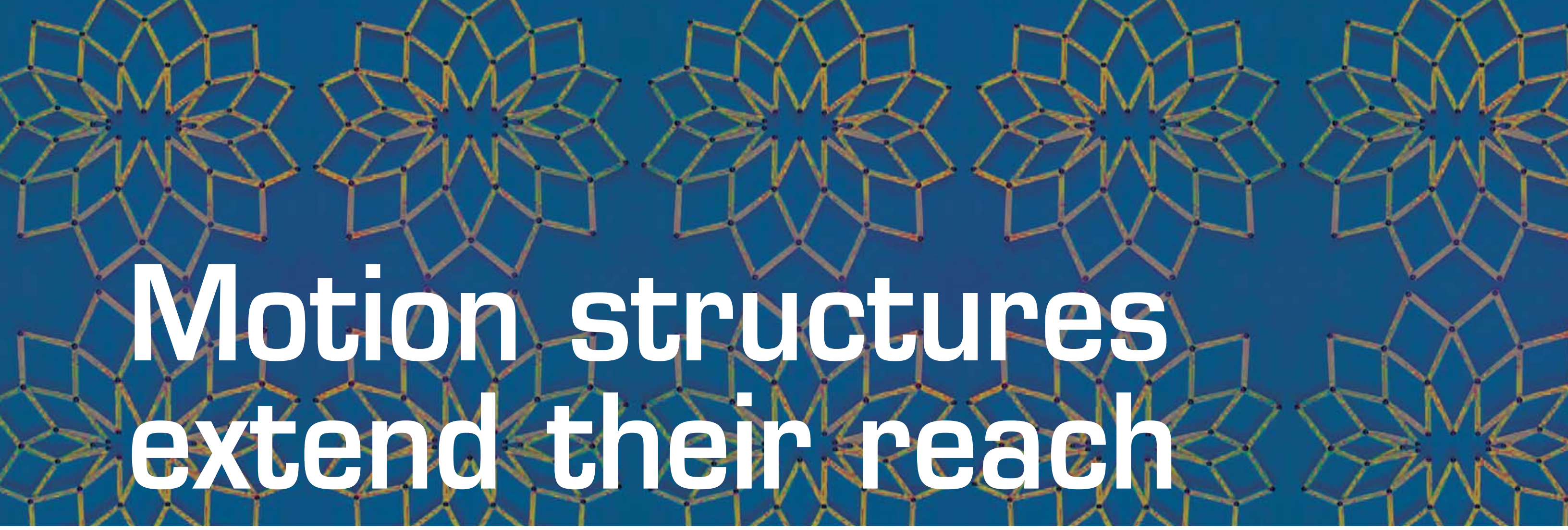

Motion structures extend their reach

Motion structures are assemblies of resistant bodies connected by movable joints. Unlike conventional structures, they are designed to contain internal mobilities that allow large shape transformations to satisfy practical requirements. Their applications range from satellite solar panels and space antennas to shelters, exhibition stands, and medical implants. We introduce the characteristics of motion structures, the principles used to create such structures, and their applications. This brief survey is based on the latest research in this thriving field.

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A structure is an assembly of resistant bodies capable of bearing loads. In general, it must be rigid and no internal mobility or relative motions among its members are allowed. However, a family of unconventional structures exists, from common household items such as umbrellas and foldable chairs to solar panels of spacecraft and leaves of hornbeam trees, that is capable of large geometrical transformation. These systems are commonly known as deployable structures and fall into two categories. The first category is the deformable structure. It is characterized by the fact that the overall strain energy of the structure varies during expansion or folding. Typical engineering examples include the storable tubular expandable member (STEM), an aerial for spacecraft similar to a carpenter's tape, and cardiovascular stents, a type of medical device for treatment of blocked or constricted blood vessels. The second category is essentially mechanism. Deployment is executed by activation of an internal mechanism carefully designed either artificially or by nature. Retractable roofs for sports facilities, the wings of beetles, and a toy called the Hoberman sphere belong to this second category.

This review focuses on the second category. Because of their internal mobilities, the term 'motion structures' is used to describe this branch of the deployable structure family. To understand the behavior of motion structures, it is necessary to review the basic building block of such assemblies: mechanisms.

A mechanism is a combination of a small number of rigid bodies (a kinematic chain) connected by movable joints (kinematic pairs) for the purpose of transmitting motion[1]. The mobility of a mechanism is the number of independent parameters that must be used to bring it into a particular position. It is generally determined by the Kutzbach criterion that has the following form for three-dimensional mechanisms:

$$m = 6(n - 1) - 5j_1 - 4j_2 - 3j_3 - 2j_4 - j_5 \qquad (1)$$

where m is the mobility of the mechanism, n is the number of links, and j_i is the number of joints having i degrees of freedom. In two dimensions, it becomes:

$$m = 3(n - 1) - 2j_1 - j_2 \qquad (2)$$

In some cases, m can become zero or less. This generally indicates that the motion is impossible and the mechanism is in fact a structure (statically determinate or indeterminate). However, there

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are exceptions because the Kutzbach criterion only considers the topological features of an assembly. Certain geometrical conditions of an assembly could lead to mobility even when m is zero or negative. This type of mechanism is called an 'overconstrained mechanism'.

Retractable roofs for sports facilities, the wings of beetles, and a toy called the Hoberman sphere are all motion structures

Most commonly used joints are called lower pairs, i.e. joints with surface contact such as a hinge (turning pair, revolute joint, or simply revolute, denoted by R), slider (a prismatic pair, denoted by P), or ball joint (denoted by S). A mechanism with only lower pairs is called a 'linkage'†. When a number of basic linkages are assembled together in such a way that the mobilities of each linkage are retained, a motion structure is born. For example, an umbrella is a combination of a number of linkages, each of which has three revolutes and one slider. The slider is the controlling parameter of its motion and it is therefore a motion structure with $m = 1$.

Among the many types of joints, revolutes are regarded as simple to make, easy to maintain, and having robust performance. They are therefore the most widely used in motion structures.

Planar motion structures with revolute joints

The basic linkage for construction of planar motion structures with revolutes (hinges) is the planar 4R linkage. Connecting the linkages side by side creates a motion structure as shown in Fig. 1. It is much more demanding to make a motion structure from a closed loop of 4R linkages. The first recorded attempt goes back to Kempe[2]. In 1878, he connected two 4R linkages to form a mobile assembly. In total, he reported six such cases. Fig. 2 shows an example of case 5 of the Kempe linkage. Note that for the Kempe linkage, $m = -3$ according to the Kutzbach criterion. But Kempe proved that his linkage has mobility arising from its particular geometry. The Kempe linkage can be also regarded as a closed chain of four scissor pairs. This naturally leads to the question of whether other mobile arrangements exist.

In 1990, Hoberman[3] discovered that if three joint locations of two plates in a scissor pair are identical, the central angle, α, sustained by the lines linking two respective end joints, remains the same when the scissor pair opens and closes (Fig. 3). He therefore declared that it was possible to build mobile closed chains of many pairs as long as the sum of the central angles is 2π, though strictly speaking, symmetry must exist in the arrangement to avoid mismatch when the first pair meets the last and forms a closed loop. This concept is the base of a well-known toy: the Hoberman flight ring (Fig. 4), which can be acquired from any good toy shop.

A more general solution has been obtained by You and Pellegrino[4], leading to the formation of a family of large foldable bar

†This definition follows that found in reference[1]. However, it is not exact and other definitions exist.

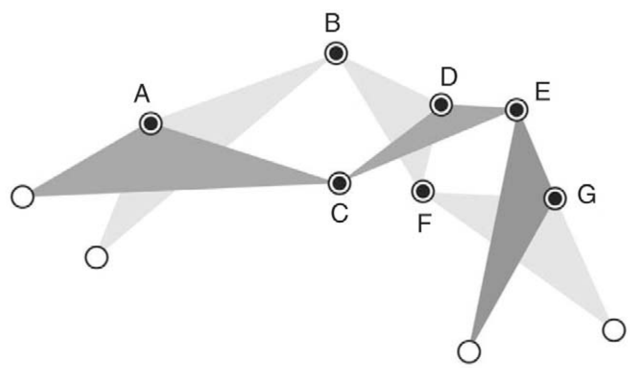

Fig. 1 A planar mobile motion structure made from two 4R linkages ABDC and DEGF. It is also called an assembly of scissor pairs because of the scissor-like beams used.

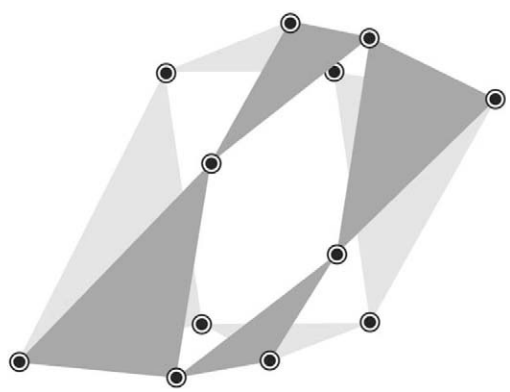

Fig. 2 Case 5 of the Kempe linkage.

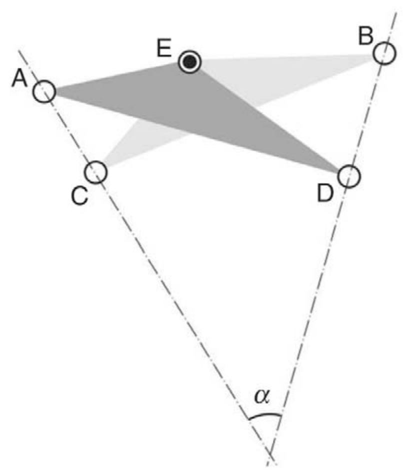

Fig. 3 A pair of identical beams forming a scissor pair. The central angle, α, is constant.

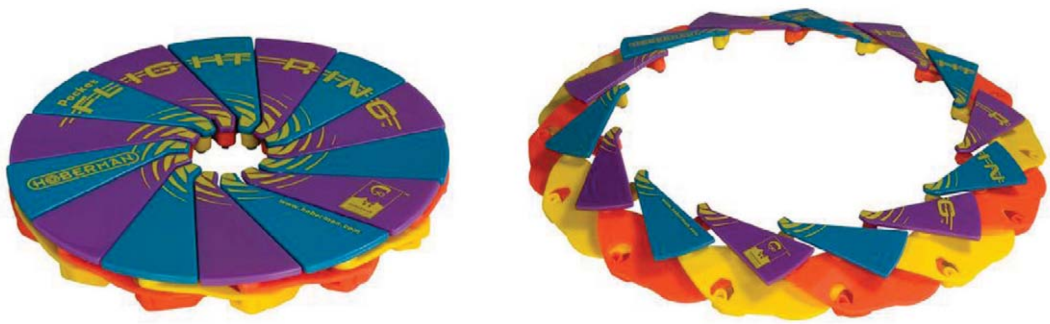

Fig. 4 Expansion of a Hoberman flight ring.

structures in which many planar parallelogram 4*R* linkages can be tessellated (Fig. 5). The structure was intended to be a supporting frame for expandable roof covers. Jensen and Pellegrino[5], and then Luo *et al.*[6], later demonstrated that a complete cover can be formed by carefully selecting the shape of the rigid plates (Fig. 6). Although the motion pattern is similar to that of an iris camera shutter, there is no sliding involved. The plates rotate about each other via the revolutes.

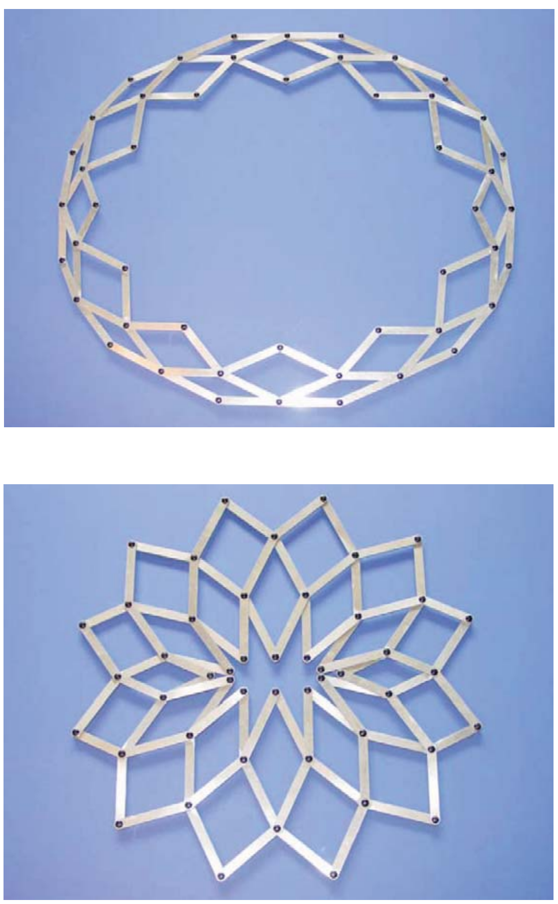

Fig. 5 Expanded and folded configurations of a mobile foldable bar structure in which the rigid plates of earlier diagrams are replaced by rigid bars.

Spatial motion structures with revolute joints

A large number of spatial motion structures can be constructed using scissor pairs. The earliest was probably a model of a travelling theatre by Pinero[7]. More recent examples can be found in structures by Escrig[8], Gantes[9], Sanchez-Cuenca[10], and Pellegrino and You[11] for applications including swimming pool covers, exhibition stands, and frames that support aerospace antenna reflectors. However, some of the motion structures mentioned here are not real mechanisms. Only in the fully folded and expanded configurations do the structural components fit together without strain. The expansion causes a small snap-through of certain members, which may sometimes be advantageous because the structures then self-lock without additional latches. On the other hand, the selection of material becomes crucial to avoid excessive strain. In kinematically mobile designs, the scissor pairs are placed so that they remain planar during deployment, i.e. four end revolutes and the mid revolute of a scissor pair remain parallel throughout deployment. Thus, they are akin to planar motion structures. One example is the Hoberman expandable sphere in which all the scissor pairs are arranged on great circles of a sphere and they remain so during expansion[3].

Although no direct reference to true three-dimensional linkages are made in publications concerning spatial structures made from scissor pairs, a close examination reveals that three-dimensional linkages do indeed appear in these assemblies. For example, the ring structure shown in Fig. 7, which can fold up radially into a compact bundle, contains a number of closed-chain 6R Sarrus linkages. One of them consists of links (a) to (f). Thus a more appropriate approach is to use existing three-dimensional linkages as building blocks for construction of motion structures.

For a closed-chain three-dimensional linkage with only revolute joints, when $m = 1$, n and j_1 (which must be the same) are 7, according to the Kutzbach criterion. It has proven immensely difficult geometrically to construct practical spatial motion structures with one internal mobility unless n and j_1 are even and less than 7, i.e. the linkages need to be exceptions to the Kutzbach criterion. As Hunt has observed[12], the most common and most useful three-dimensional mechanisms to be discovered date back many years and are overconstrained linkages.

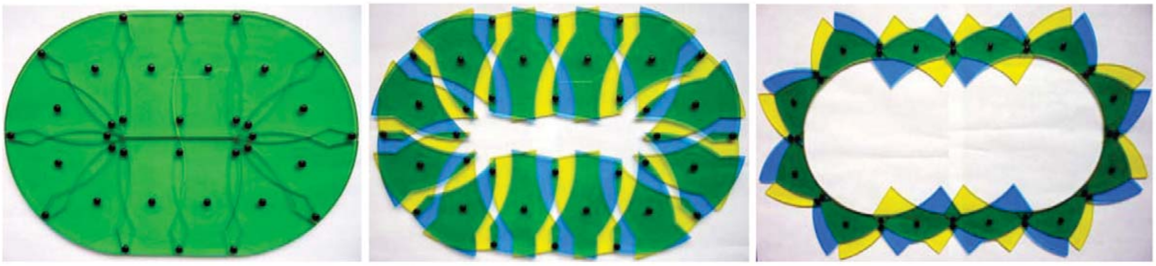

Fig. 6 Expansion of a mobile planar structure that can be used as a retractable roof. The expanded configuration is shown at a smaller scale than the folded configuration.

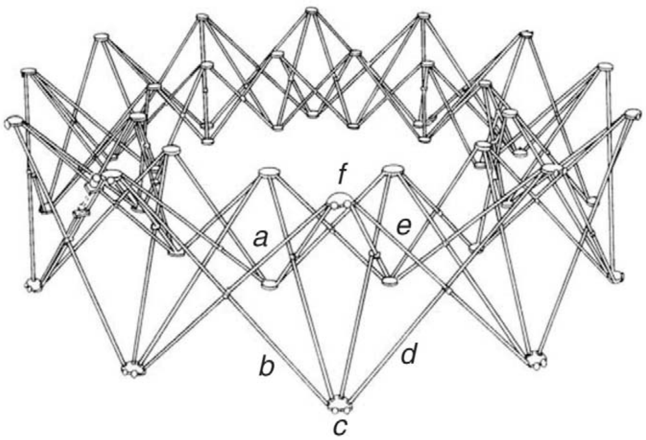

Fig. 7 A deployable ring structure for supporting a reflective mesh reflector at the centre.

The smallest number of links required to form a meaningful three-dimensional closed-chain linkage with revolutes is four. There are two types of three-dimensional 4*R* linkages. The first, shown in Fig. 8, is the spherical 4*R* linkage in which the axes of the revolutes meet at one point. Kovács *et al.*[13] constructed a class of expandable polyhedral structures based on the linkage, which simulate the swelling motion of the cowpea chlorotic mottle virus. The other 4*R* linkage is the Bennett linkage[14] (Fig. 9), in which the axes of the revolutes are neither parallel nor concurrent. The links shown in Fig. 9 are perpendicular to the axes of revolutes at both ends, and thus they represent the shortest distances between the axes. In spite of being regarded by mechanical engineers as "probably one of the most useless of the known spatial linkages"[1], this linkage is actually quite suitable for construction of motion structures. In kinematics, a number of other overconstrained linkages, such as the Goldberg 5*R* and 6*R* linkages, the Bennett-joint 6*R* linkage, and the Wohlhart double-Goldberg linkages, have been found by combining or merging several Bennett linkages (termed as addition and subtraction by Goldberg)[15]. This approach has laid the foundation for construction of motion structures based on the Bennett linkage.

Motion structures are constantly evolving to overcome engineering problems in all walks of life

In 2005, Chen and You[16] reported the discovery of a family of motion structures based on the Bennett linkage. An arch is shown in Fig. 10 that contains many Bennett linkages (similar to Fig. 9) nested within each other. The assembly has a single degree of freedom. The concept can be extended to form structures that deploy into a tower or a helical profile. Moreover, the Bennett linkage can be built such that it folds into a compact bundle. This is realized by replacing the links with rigid pieces that do not span the shortest distances between the axes of the respective revolutes that they connect. These rigid pieces are designed so that they collapse together completely in the folded configuration[17]. An arch frame to this effect is shown in Fig. 11. It is

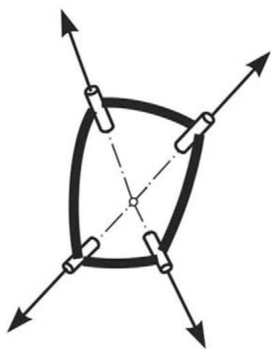

Fig. 8 A spherical 4R linkage.

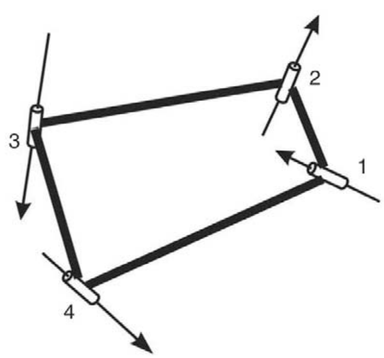

Fig. 9 A 4R Bennett linkage.

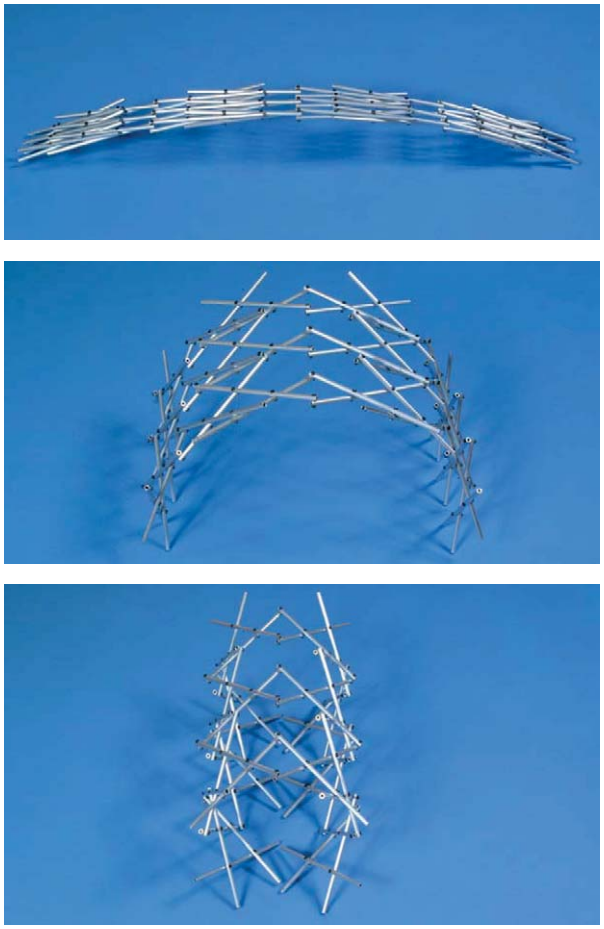

Fig. 10 Deployment sequence of a motion structure based on the Bennett linkage. It has an arch profile consisting of three interlinked arches.

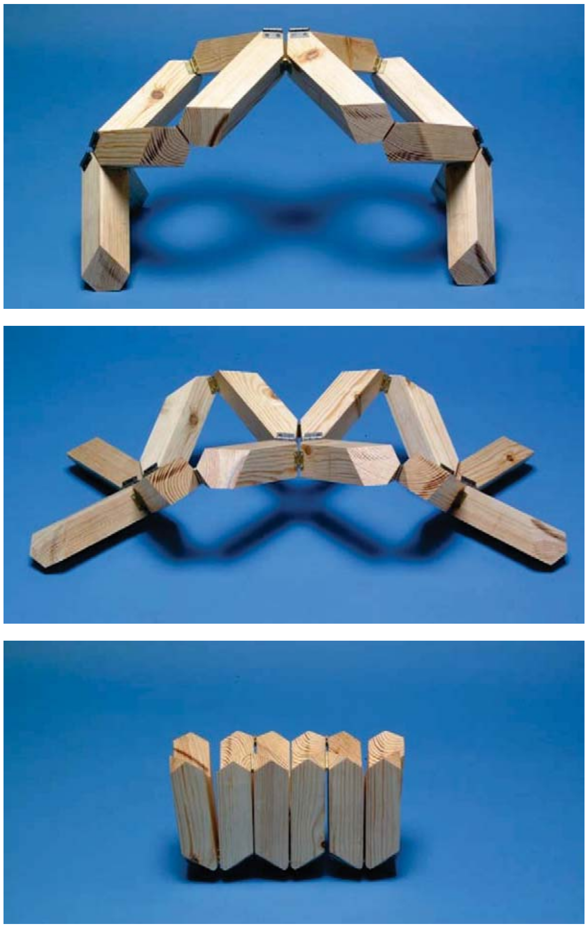

Fig. 11 A Bennett-linkage-based arch capable of compact folding.

interesting to note that both arches (Figs. 10 and 11) are based on the same framework but the folded configurations are different.

The key to the design of Bennett-linkage-based structures is that the same deployable units are repeated again and again in a structural assembly. In fact, this approach is not new. A type of deployable boom consisting of a series of foldable cubes was proposed by NASA engineers back in 1988[18]. A complete study of all the possibilities of foldable cubes should be credited to Britt and Lalvani[19]. With these basic deployable units, Chen[20] has suggested the use of mathematical tiling and patterns as a tool to build large motion structures. However, it is still not very practical because of the large number of possibilities involved and both the geometrical and algebraic analysis are daunting.

Origami structures

There is another family of motion structures that is closely linked with the three-dimensional linkages using revolute joints, namely origami structures. Origami is the art of folding paper along a small set of predetermined creases (revolutes) to make intricate two- or three-dimensional designs. Common origami patterns contain many vertices where the creases meet. Hence, around each vertex, the paper panels form a spherical linkage in which all of the axes of the revolutes also meet at one point (Fig. 8). This is particularly true if the panels are not allowed to stretch or to bend, a restrictive form of origami called 'rigid origami'. Fig. 12 shows an example.

Origami patterns have been used to create motion structures

Origami motion structures also exist in nature. The wings of beetles and the leaves of hornbeams and beeches are found to mimic certain origami patterns[21,22]. Origami patterns have also been used to create motion structures. The most well known example is the Miura-Ori, a map-folding pattern that allows simultaneous expansion in all directions through pulling of the diagonal corners of a folded paper[23]. This pattern has been used for deployable solar panels in space. A family of solid-surface deployable antennas, reported in 1996[24], were

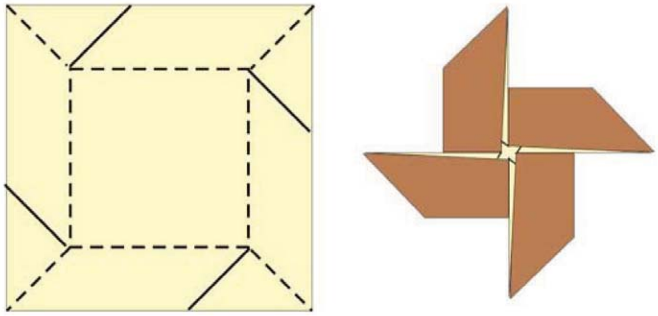

Fig. 12 Origami pattern for a windmill. The solid and dashed lines represent hill and valley creases, respectively. It has four vertices, each of which is a spherical linkage.

inspired by a wrapping pattern of thin membrane developed in the 1960s. In addition to folding of planar objects, origami patterns have also been used in three dimensions. Foldable cylinders with triangulated patterns for space booms[25] and an origami stent graft (Fig. 13)[26] for the treatment of abdominal aortic aneurysms are two examples. In both cases, the folding patterns are not real mechanisms because they could not be folded if the panels were completely rigid. However, the geometrical distortion within the panels has been localized and minimized. It has been found to be extremely challenging to identify rigid folding patterns with true mobility.

Further developments

In the last few years, it has been observed that the boundary between motion structures and deformable deployable structures is gradually blurring. Flexible structural components have been introduced to

Fig. 13 Models of an origami stent graft that can be used in the treatment of oesophageal cancer. The models are made from stainless steel tubes to obtain good geometrical compatibility during expansion, though in reality much more flexible materials are used.

mobile assemblies to create bistable self-locking structures. Smart structures with embedded sensors and actuators have begun to appear that are capable of responding to the environment in which they function. More recently, thin-walled structures with origami patterns have been developed in order to achieve better energy absorption ability when they collapse under impact loading. It is predicted that such structures may be used as microstructural units for ultralight materials with superior thermal and mechanical properties.

Motion structures have long been in use and remain at the forefront of engineering endeavor. They are constantly evolving to overcome engineering problems that exist in all walks of life. mt

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