Method to Simulate Design Earthquake Motions

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Earthquake motions for structure design

- Using observed motion
- Stochastic model of amplitude and phase spectra
- Based on the dynamics of elastic body
- Empirical simulation model
- Design response spectrum compatible EM
- EM phase using nonlinear structural design

Introduction of Fourier analysis and stochastic modeling of earthquake motions

Modeling of amplitude and phase

Introduction of Fourier analysis

- Using inverse Fourier transformation

$$y(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} Y(\omega)d\omega \frac{1}{2\pi}\int_{-\infty}^{\infty} A(\omega)\exp(i\phi(\omega))e^{i\omega t}d\omega$$

$Y(\omega)$: *Fouier transform of* $y(t)$

$A(\omega)$: *amplitude spectrum*

$\phi(\omega)$: *phase spectrum*

Fourier Inverse Transform

$$y(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} A(\omega)\exp(i\phi(\omega))e^{i\omega t}d\omega$$
$$= \int_{-\infty}^{\infty} A(\omega)\cos(\omega t + \phi(\omega))df + i\int_{-\infty}^{\infty} A(\omega)\sin(\omega t + \phi(\omega))df$$

Taking into account the facts that the amplitude spectrum is symmetric and the phase spectrum is asymmetric with respect to , we can get

$$y(t) = \int_{-\infty}^{\infty} A(\omega)\cos(\omega t + \phi(\omega))df$$

Discretization

It is necessary for numerical analyses to use finite sampling function given by

$$G(\omega) = \delta(\omega) + \sum_{l=1}^{N/2}\{\delta(\omega-\omega_l) + \delta(\omega+\omega_l)\}$$

And considering the following formula

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} A(\omega)e^{i(\omega t+\phi(\omega))}\{\delta(\omega-\omega_l) + \delta(\omega+\omega_l)\}2\pi df$$
$$= 2A(\omega_l)\cos(\omega_l t + \phi(\omega_l))$$

Discretization

Substituting these formula into the inverse transformation formula we can obtain

$$y(t) = A(0) + \sum_{l=1}^{N/2} 2A(\omega_l)\cos(\omega_l t + \phi(\omega_l))$$

Any time histories can be approximated by a finite sum of cosine time functions

Amplitude and phase spectra of earthquake motion

$$y(t) = A_1\cos(2\pi f_1(t+t_1)) + A_2\cos(2\pi f_2(t+t_2)) + \ldots$$
$$+ A_n\cos(2\pi f_n(t+t_n)) + \ldots$$

f_n : *nth frequency*

A_n : *nth amplitude* $\rightarrow$ *amplitude spectrum*

t_n : *nth time lag*

$2\pi f_n t_n$: *nth phase* $\rightarrow$ *phase spectrum* : ϕ_n

Simple case examples

$$y(t) = A_1 \cos(2\pi f_1 (t + t_1)) + A_2 \cos(2\pi f_2 (t + t_2))$$

$case\,1: A_1 = A_2 = 1.0,\ f_1 = f_2 = 1Hz$

$t_1 = 0.25, \quad t_2 = -0.25_{\text{sec}}$

c1

$case\,2: A_1 = A_2 = 1.0,\ f_1 = 1.0,\ f_2 = 1.1$

$t_1 = 0.25, t_2 = -0.25$

c2

Complicated example

- Amplitude is defined by normal distribution with mean=0 and standard deviation=5gal
- In the phase $2\pi f_n t_n$, $f_n t_n$ is assumed to be expressed by uniform random number between 0 to 1
- Number of summation terms is change from 10 to 1000
- Time interval is 0.01 sec and duration is 10 sec

Sample 3

Shape function

$0 \le t \le T_b \qquad E(t) = (t/T_b)^2$

$T_b < t \le T_c \qquad E(t) = 1$

$T_c < t \le T_d \qquad E(t) = \exp(-a(t - T_c))$

$T_d < t \qquad E(t) = 0.1$

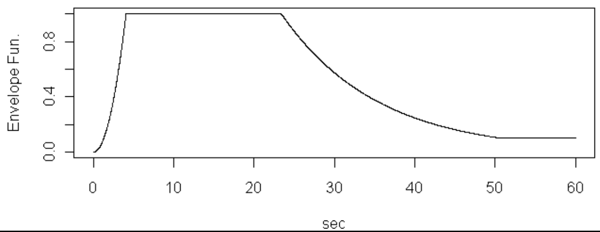
$T_d = 10^{0.31M - 0.774}$

$T_b = (0.12 - 0.04(M - 7))T_d$

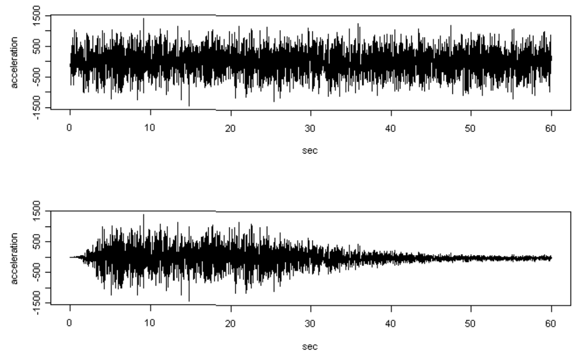
$T_c = (0.50 - 0.04(M - 7))T_d$

$a = -\ln(0.1)/(T_d - T_c)$

M : *Earthquake Magnitude*



Stationary and Non-stationary time history of accelerations

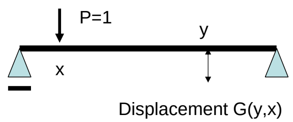


Introduction of simulation of earthquake motions based on elasto dynamics

Similarity law and derived simple formula

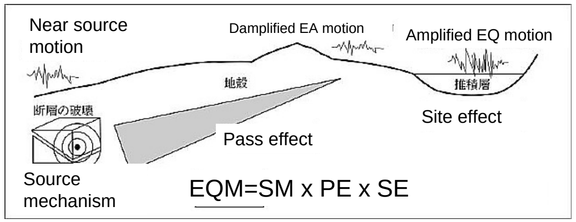
Based on the dynamics of elastic body

- Partial differential equation controlling wave propagation in elastic medium
- Green function of this equation

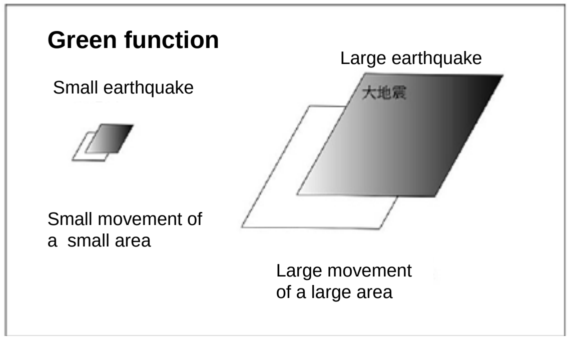


Displacement u(t) at y for a given time history of force P(t) at x is calculated by u(t)=G(y,x)P(t)

Mechanism of wave propagation



Source mechanism



Similarity Law

$$M_0 = \mu D_0 A$$

$$M_0 \sim A^{3/2} \quad L \approx 2W \quad D_0 \sim L$$

$$\text{Large Event}: L_L \ W_L \ D_{0L}$$

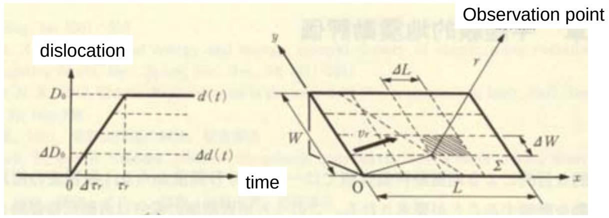
$$\text{Small Event}: \Delta L \ \Delta W \ \Delta D_0$$

$$L_L / \Delta L = W_L / \Delta W = D_{0L} / \Delta D_0 = n_L = n_S = n_D = n$$

$$n = \left(\frac{M_L}{M_s} \right)^{\frac{1}{3}}$$

Uniform rupture of faults

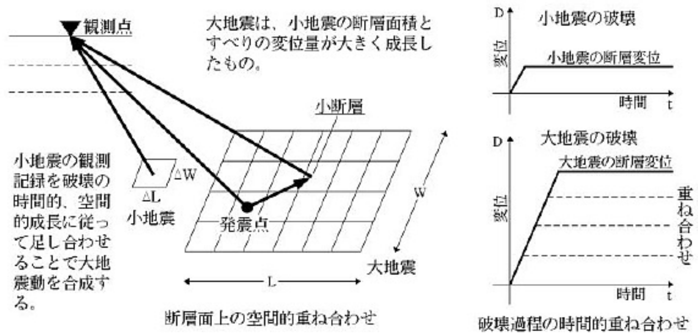
$$u_L(t) = \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} \frac{r_s}{r_{ij}} u_S \left(t - t_{ij} - (k-1)\frac{\tau_r}{n} \right)$$

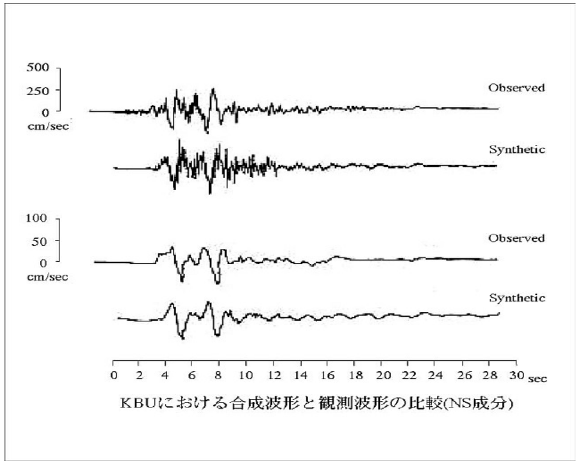


Simulation of earthquake motion for a large event

- Calculate Green function based of wave propagation equation in elastic medium
- Sum up Green function taking into account similarity rule between the samll event and a large event
- Difficult to obtain Green function taking into account complex path and local site effect

Simulation of earthquake motion using observed small event motion





KBUにおける合成波形と観測波形の比較(NS成分)

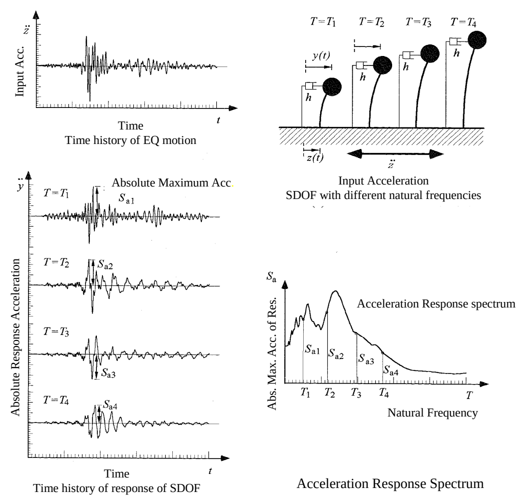
Concept of Seismic Design

For the case of Elastic design

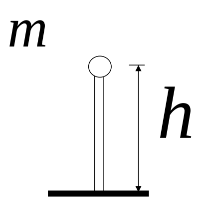
Introduction of seismic design

- Concept of response spectra
- How to use response spectra for earthquake proof design of structures
- Concept of yield strength demand spectrum
- How to use the yield strength demand spectrum for earthquake proof design of structures

Concept of Response Spectrum



Concept of aseismic design for a linear structure



$$F = mS_a(T,h)$$

Bending moment at colum foot

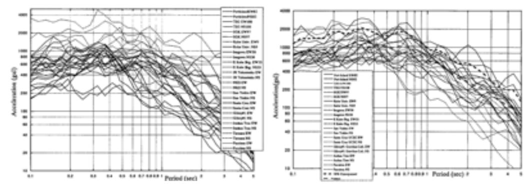
$$M = Fh = mhS_a(T,h)$$

$$M \le M_{\max}$$

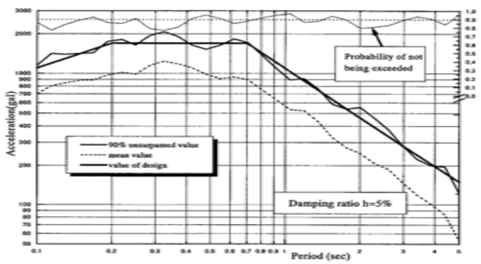
Earthquake data and calculated acceleration response spectra for near source region

Table1. Near-source seismic records from recent earthquakes

No	Earth-quake	Name of seismic record	Max. Acc. (gal) NS	Max. Acc. (gal) EW	Latitude	Longitude	Equivalent hypocentral distance	Shortest Distance to Fault	Ground level of observation
1	Hyogoken-Nanbu	Port Island	679.8	302.6	34.670	135.208	11.64	3.24	GL-83
2		Takasago Power Station	86.0	109.3	34.753	134.783	32.75	27.08	GL-100
3		SGK.KansaiElec.Power Co.	293.9	319.8	34.743	135.442	34.57	24.65	GL-97.0
4		Roko(Kobe University)	272.0	306.5	34.725	135.240	14.99	6.90	GL-9.5
5		Inagawa	185.3	200.4	34.836	135.427	38.03	25.03	GL-30
6		Great Bridge of East Kobe	445.9	425.3	34.707	135.296	20.00	12.38	GL-33
7		Takarazuka	683.6	600.9	34.809	135.344	29.93	16.88	GL0.0
8		New Kobe substation	510.7	584.2	34.731	135.250	16.52	7.53	GL0.0
9	Coyote Lake	San Ysidro	314.6	408.8	37.026	121.484		1.0	GL0.0
10	Loma Prieta	Santa Cruz UCSC	433.1	401.5	37.00	122.06	18.01	12.19	GL0.0
11		Gilroy#1-Gavilan Coll.	426.6	433.6	36.973	121.572	26.56	12.21	GL0.0
12	Landers	Joshua Tree fire station	268.3	278.4	34.131	116.314	16.90	10.79	GL0.0
13	Northridge	Tarzana Cedar Hill	970.7	1744.5	34.160	118.534	20.63	17.87	GL0.0
14		Pacoima Kagel Canyon	424.2	295.2	34.288	118.375	18.73	8.98	GL0.0

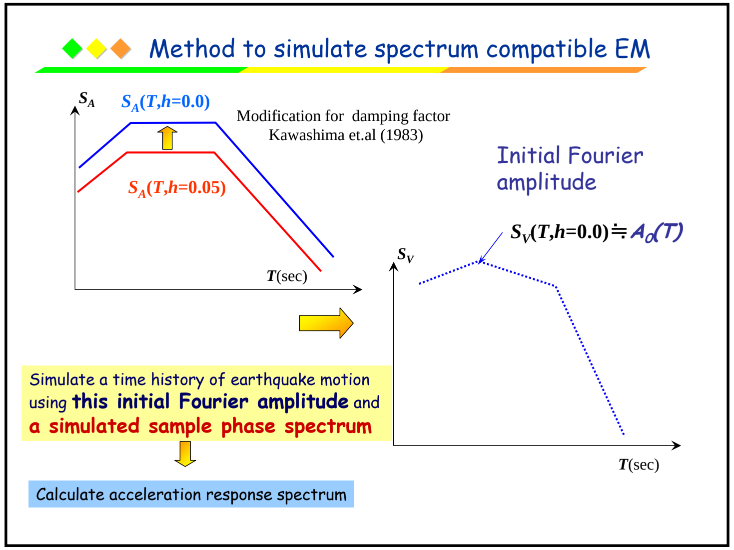


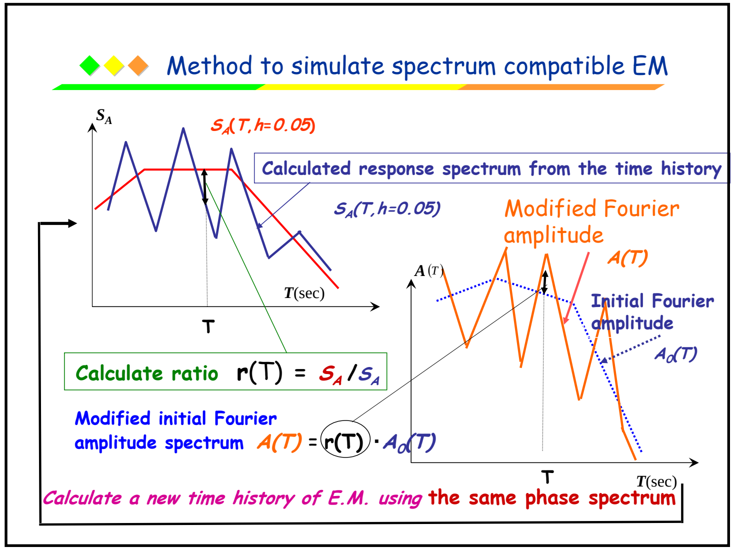
Design response spectrum for intra-plate earthquake

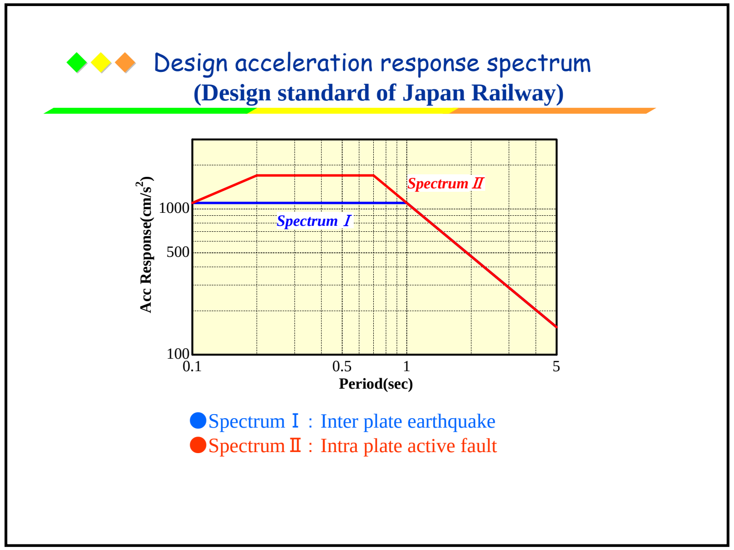


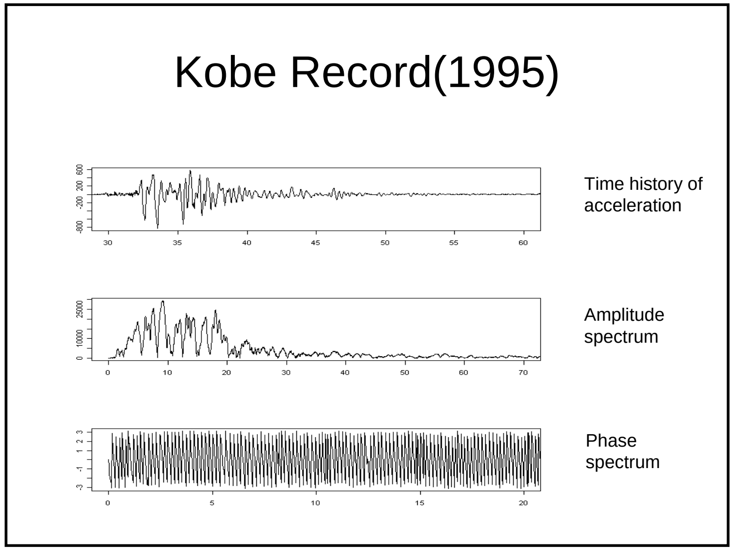
Simulation of Design Spectrum Compatible Earthquake Motions

Inter and intra plate design spectra

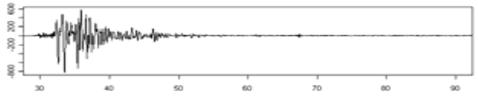

Method to simulate spectrum compatible EM
SA
SA(T,h=0.0)
Modification for damping factor
Kawashima et.al (1983)
SA(T,h=0.05)
T(sec)
Initial Fourier
amplitude
SV(T,h=0.0)≒A0(T)
SV
T(sec)
Simulate a time history of earthquake motion
using this initial Fourier amplitude and
a simulated sample phase spectrum
Calculate acceleration response spectrum


Method to simulate spectrum compatible EM
SA
SA(T,h=0.05)
Calculated response spectrum from the time history
SA(T,h=0.05)
Modified Fourier
amplitude
A(T)
A(T)
Initial Fourier
amplitude
A0(T)
T(sec)
T
Calculate ratio r(T) = SA / SA
Modified initial Fourier
amplitude spectrum A(T) = r(T)・A0(T)
T
T(sec)
Calculate a new time history of E.M. using the same phase spectrum

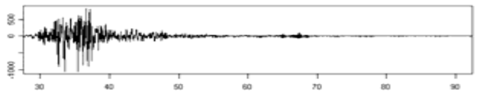

Design acceleration response spectrum
(Design standard of Japan Railway)
Acc Response(cm/s2)
1000
500
100
Spectrum Ⅱ
Spectrum Ⅰ
0.1
0.5
1
5
Period(sec)
●Spectrum Ⅰ : Inter plate earthquake
●Spectrum Ⅱ : Intra plate active fault


Kobe Record(1995)
Time history of
acceleration
Amplitude
spectrum
Phase
spectrum

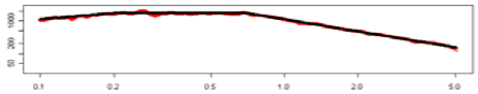
For the case of Kobe Record



Original time history

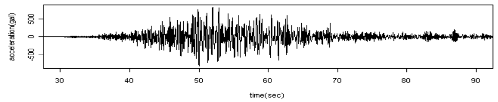


Response spectrum compatible TH

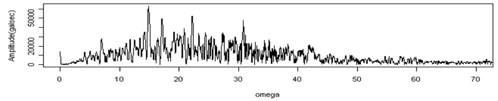


Comparison between target and recalculated response spectrum

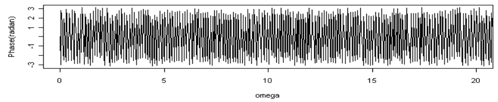
Kushiro Record(1994)



Time history of acceleration

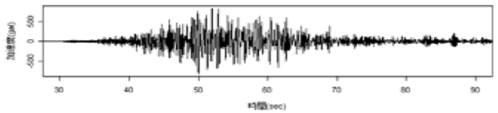


Amplitude spectrum

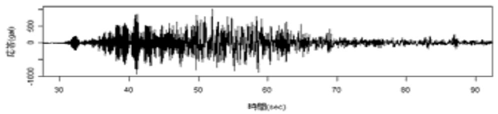


Phase spectrum

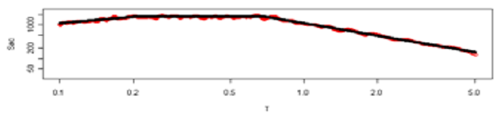
For the case of Kushiro Record



Original time history



Response spectrum compatible TH

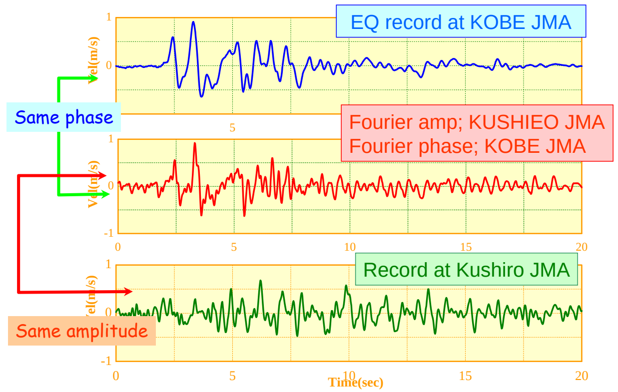


Comparison between target and recalculated response spectrum

Modeling of Phase Spectra for Simulation of Design Earthquake Motions

Single Source Assumption

Effect of phase on earthquake ground motion



◆ Definition of group delay time (GDT)

Group delay time

$$t_{gr} = \frac{d\phi(\omega)}{d\omega}$$

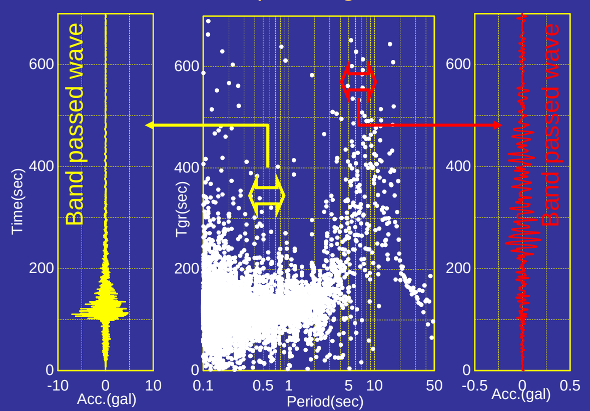
$$f(t) \Leftrightarrow A(\omega)\exp\{i\phi(\omega)\}$$

- **Mean value : Arrival time of energy of E.M.**
- **Standard deviation : Duration of E.M.**

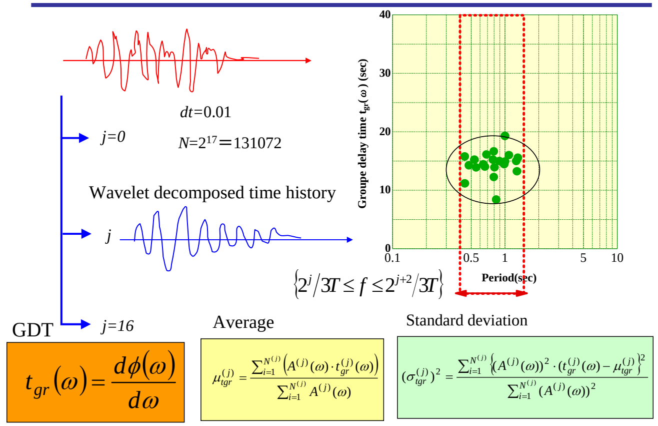
Modeling of GDT is easier than the direct modeling of phase spectrum $\phi(\omega)$

Role of Group Delay Time

Samples of tgr



Mean group delay time and its standard deviation - *Using Mayer Wavelet* -



Decompositon of GDT into source, path and local site effects

$$A(\omega)e^{i\phi(\omega)}$$
$$= A^{(S)}(\omega)\cdot A^{(P)}(\omega)\cdot A^{(L)}(\omega)\cdot \exp\left\{i\left(\phi^{(S)}(\omega)+\phi^{(P)}(\omega)+\phi^{(L)}(\omega)\right)\right\}$$

$$t_{gr}(\omega)=\frac{d\phi}{d\omega}$$

$$t_{gr}^{(j)}(\omega)=t_{gr}^{(j)(S)}(\omega)+t_{gr}^{(j)(P)}(\omega)+t_{gr}^{(j)(L)}(\omega)$$

$$\mu_{tgr}^{(j)}=\mu_{tgr}^{(j)(S)}+\mu_{tgr}^{(j)(P)}+\mu_{tgr}^{(j)(L)}$$

$$(\sigma_{tgr}^{(j)})^2=(\sigma_{tgr}^{(j)(S)})^2+(\sigma_{tgr}^{(j)(P)})^2+(\sigma_{tgr}^{(j)(L)})^2$$

Regression equation of GDT

$$\mu_{tgr}^{(j)}(\omega)=\alpha_1^{(j)}\cdot 10^{0.5M}+\beta_1^{(j)}\cdot R+\gamma_1^{(j)}\cdot H+\kappa_1^{(j)}$$

$$(\sigma_{tgr}^{(j)})^2=\alpha_2^{(j)}\cdot 10^{M}+\beta_2^{(j)}\cdot R^2+\gamma_2^{(j)}\cdot H^2+\kappa_2^{(j)}$$

M: Earthquake magnitude
R: Hypocenter distance
H: Depth of surface ground

◆◆◆ Number of data used for analysis

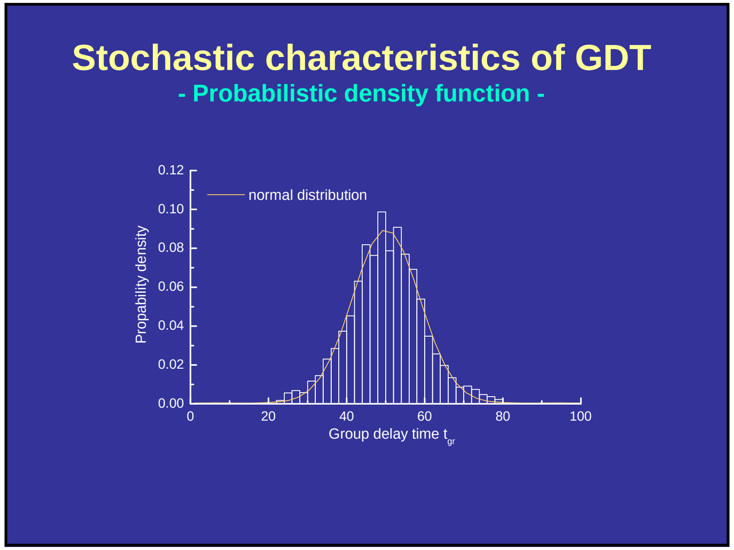
Table 1 Number of data used for analysis

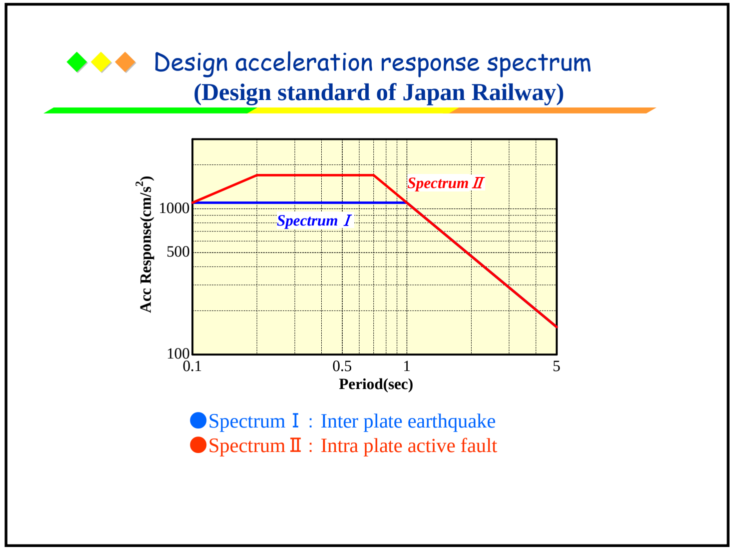
Coefficients	Number of data
α	588
β	786
γ,κ	1618

Regression coefficients

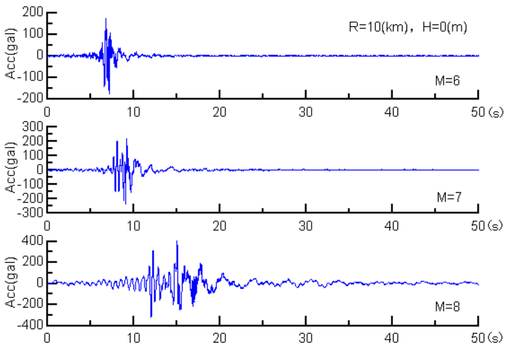
Table 2 Result of regression analyses

j	$\mu_{tgr}^{(j)}$				$(\sigma_{tgr}^{(j)})^2$			
	α_1	β_1	γ_1	κ_1	α_2	β_2	γ_2	κ_2
8	1.359E-03	0.362	4.026E-03	0.250	1.206E-05	4.163E-03	6.408E-06	142.599
9	1.453E-03	0.326	4.404E-03	2.930	3.131E-06	3.433 E-03	7.440E-06	109.490
10	8.439E-04	0.298	3.345E-03	4.712	1.268E-06	3.480 E-03	5.918E-06	51.728
11	8.247E-04	0.286	2.180E-03	3.810	5.919E-07	3.112E-03	2.073E-06	23.115
12	6.080E-04	0.271	1.097E-03	3.442	8.891E-09	1.767E-03	4.627E-07	12.476
13	8.880E-04	0.251	6.145E-04	3.656	8.805E-08	6.257E-04	3.760E-07	11.656
14	1.143E-03	0..250	3.723E-04	3.174	8.068E-08	5.522E-04	3.724E-07	9.915
15	1.037E-03	0.246	1.553E-05	3.169	7.015E-07	9.468E-04	3.673E-07	12.376

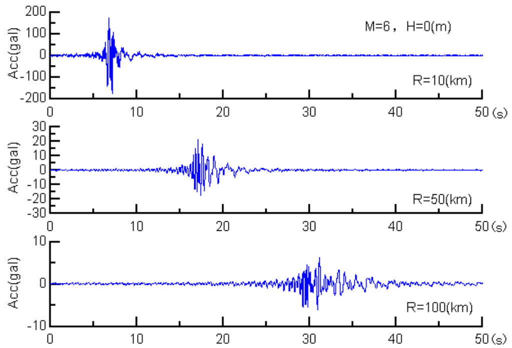




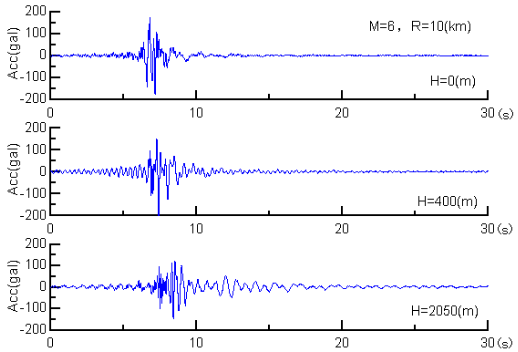
EM compatible with RS effect of magnitude



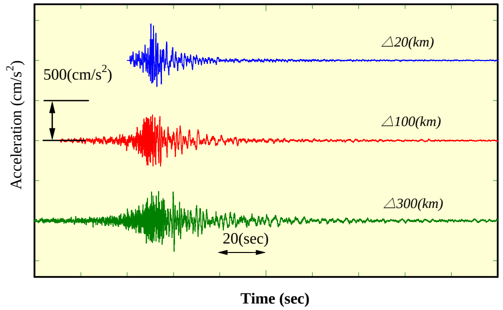
EM compatible with RS effect of hypocenter distance



EM compatible with RS
effect of surface deposit (depth)



◆◆◆ Spectrum compatible EM -Spectrum I-



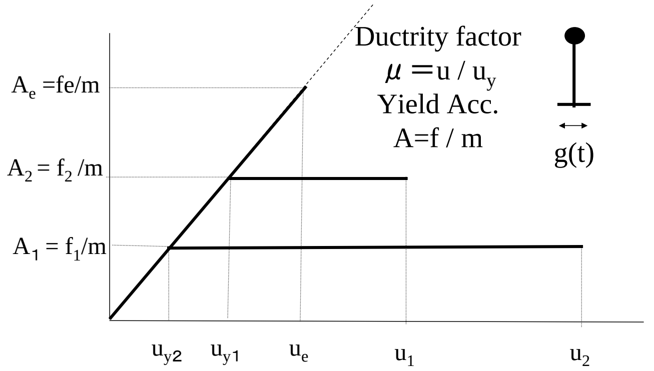
Concept of Seismic Design

For the case of nonlinear design

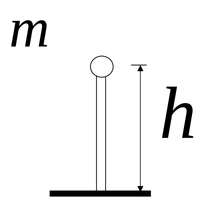
Introduction of seismic design

- Concept of response spectra
- How to use response spectra for earthquake proof design of structures
- Concept of yield strength demand spectrum
- How to use the yield strength demand spectrum for earthquake proof design of structures

Definition of yield acceleration and displacement as well as ductility factor



Concept of aseismic design for a nonlinear structure



$F = mA_y(T,h)$

Bending moment at colum foot

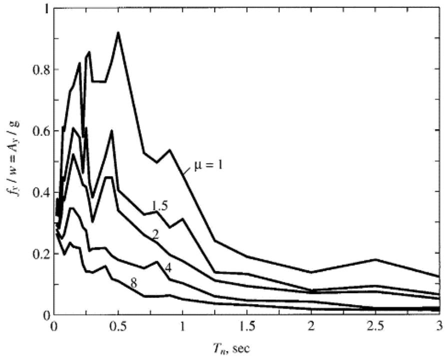
$M = Fh = mhA_y(T,h)$

$M \leq M_{\max}$

$A_e(T,h) = S_a(T,h)$

$A_1(T,h) < A_2(T,h) < A_e(T,h)$

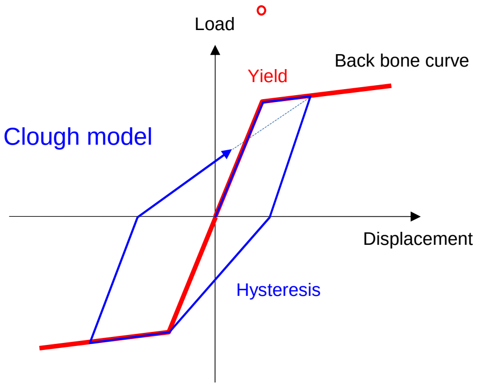
Strength demand spectra



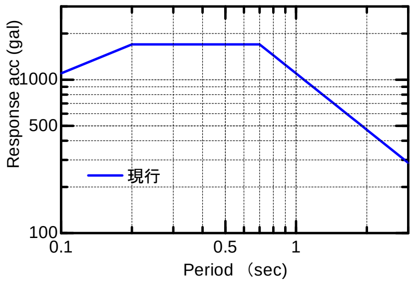
Earthquake Phase Spectrum used for Nonlinear Structural Design

Concept of worst earthquake phase

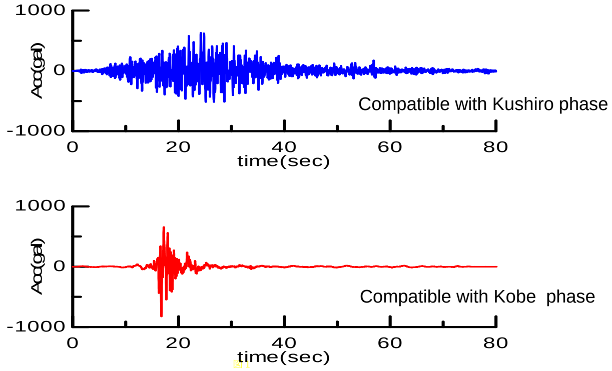
Clough Hysteresis Model



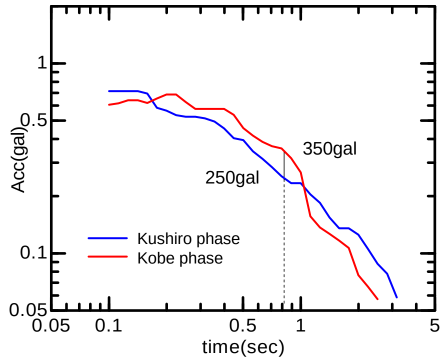
Response spectrum used for calculation



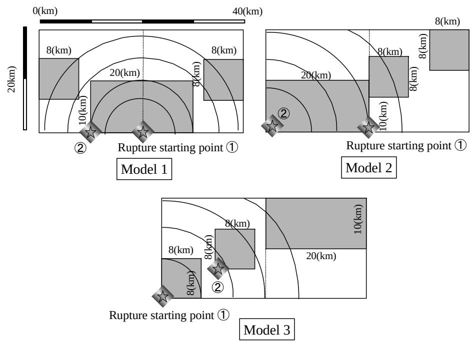
Response spectrum compatible earthquake motions



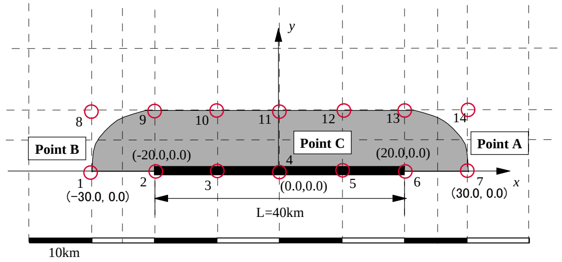
Nonlinear response spectra



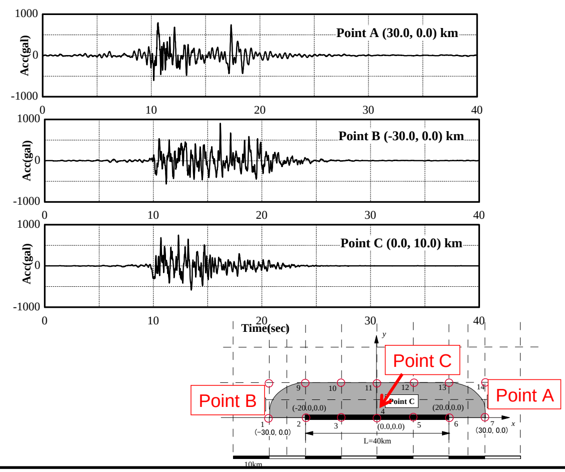
Asperity distribution used for parameter studies



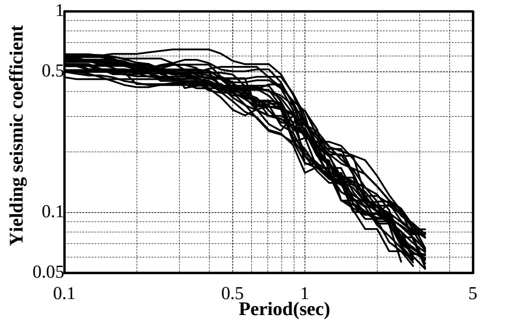
Positions to simulate earthquake motions



Example of simulated earthquake motions

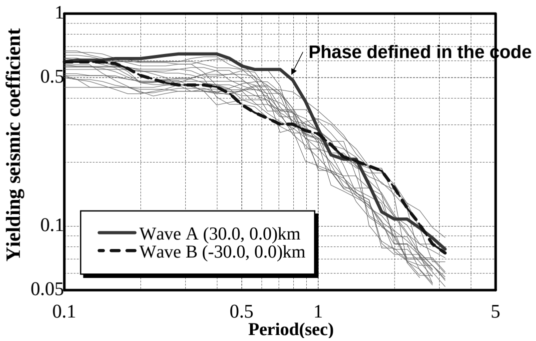


Nonlinear response spectra using simulated earthquake motion's phases



Comparison with nonlinear response spectra with those of observed earthquake motion phases

Thin line: nonlinear response spectra using observed phases



Thank you very much for your attention

A棟, B棟, イベント広場を見る

Port Island Campus of Kobegakuin University